

# 8.1

## Ratio and Proportion

- Goals**
- Find and simplify the ratio of two numbers.
  - Use proportions to solve real-life problems.

### VOCABULARY

Ratio of  $a$  to  $b$

Proportion

Extremes

Means

### Example 1 Simplifying Ratios

Simplify the ratio.

a.  $\frac{16 \text{ kg}}{800 \text{ g}}$

b.  $\frac{7 \text{ ft}}{21 \text{ yd}}$

#### Solution

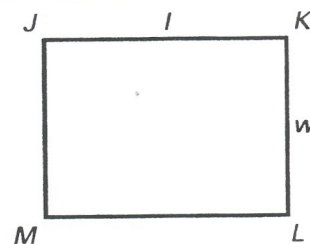
To simplify ratios with unlike units, convert to like units so that the units divide out. Then simplify the fraction, if possible.

a.  $\frac{16 \text{ kg}}{800 \text{ g}} = \frac{16 \cdot \boxed{\phantom{000}} \text{ g}}{800 \text{ g}} = \frac{\boxed{\phantom{000}}}{800} = \underline{\hspace{1cm}}$

b.  $\frac{7 \text{ ft}}{21 \text{ yd}} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

**Example 2** Using Ratios

The perimeter of rectangle  $JKLM$  is 56 centimeters. The ratio of  $JK : KL$  is 4 : 3. Find the length and width of the rectangle.

**Solution**

Because the ratio of  $JK : KL$  is  $\underline{\hspace{1cm}} : \underline{\hspace{1cm}}$ , you can represent the length of  $JK$  as  $\underline{\hspace{1cm}}x$  and the width of  $KL$  as  $\underline{\hspace{1cm}}x$ .

$$2l + 2w = P \quad \text{Formula for perimeter of rectangle}$$

$$2(\underline{\hspace{1cm}}x) + 2(\underline{\hspace{1cm}}x) = 56 \quad \text{Substitute for } l, w, \text{ and } P.$$

$$\underline{\hspace{1cm}}x + \underline{\hspace{1cm}}x = 56 \quad \text{Multiply.}$$

$$\underline{\hspace{1cm}}x = 56 \quad \text{Combine like terms.}$$

$$x = \underline{\hspace{1cm}} \quad \text{Divide each side by } \underline{\hspace{1cm}}.$$

**Answer** So,  $JKLM$  has a length of  $\underline{\hspace{1cm}}$  centimeters and a width of  $\underline{\hspace{1cm}}$  centimeters.

✓ **Checkpoint** Simplify the ratio.

1.  $\frac{5 \text{ km}}{200 \text{ m}}$

2.  $\frac{40 \text{ oz}}{5 \text{ lb}}$

**PROPERTIES OF PROPORTIONS**

**1. Cross Product Property** The product of the extremes equals the product of the means.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \underline{\hspace{1cm}} = \underline{\hspace{1cm}}.$$

**2. Reciprocal Property** If two ratios are equal, then their reciprocals are also equal.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \underline{\hspace{1cm}} = \underline{\hspace{1cm}}.$$

### Example 3 Solving Proportions

Solve the proportion  $\frac{t + 4}{8} = \frac{t}{3}$ .

$$\frac{t + 4}{8} = \frac{t}{3}$$

Write original proportion.

$$\underline{\hspace{1cm}}(t + 4) = \underline{\hspace{1cm}}t$$

Cross product property

$$\underline{\hspace{1cm}}t + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}t$$

Distributive property

$$\underline{\hspace{1cm}} = t$$

Simplify.

### Example 4 Solving a Proportion

A scale model of a car is 10 inches long and 5 inches tall. The actual car is 60 inches tall. What is the length of the actual car?

Verbal  
Model

$$\frac{\text{Length of car}}{\text{Length of model}} = \frac{\text{Height of car}}{\text{Height of model}}$$

Labels

$$\text{Length of car} = x \qquad \text{Height of car} = \underline{\hspace{1cm}} \text{ (in.)}$$

$$\text{Length of model} = \underline{\hspace{1cm}} \qquad \text{Height of model} = \underline{\hspace{1cm}} \text{ (in.)}$$

Reasoning

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Substitute.

$$x = \underline{\hspace{1cm}}$$

Multiply each side by  $\underline{\hspace{1cm}}$ .

$$x = \underline{\hspace{1cm}}$$

Simplify.

Answer So, the actual car is  $\underline{\hspace{1cm}}$  inches, or  $\underline{\hspace{1cm}}$  feet long.

✓ **Checkpoint** Solve the proportion.

$$3. \frac{7}{2} = \frac{21}{r}$$

$$4. \frac{6}{3x - 12} = \frac{4}{x}$$

**Practice A**

For use with pages 457–464

The girls' soccer team won 10 games and lost 2, and the boys' soccer team won 12 games and lost 3.

1. What is the ratio of the girls' wins to their losses?
2. What is the ratio of the boys' wins to their losses?
3. What is the ratio of the girls' wins to the total number of games played?
4. What is the ratio of the boys' wins to the total number of games played?
5. Which team had the greater winning ratio?

**Simplify the ratio.**

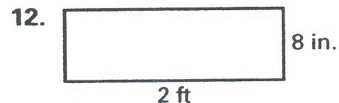
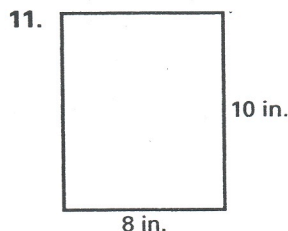
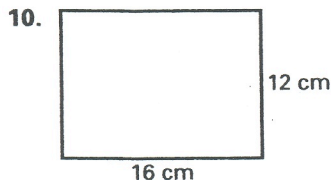
6.  $\frac{6 \text{ yards}}{12 \text{ yards}}$

7.  $\frac{14 \text{ trucks}}{7 \text{ trucks}}$

8.  $\frac{16 \text{ people}}{24 \text{ people}}$

9.  $\frac{32 \text{ meters}}{24 \text{ meters}}$

**Find the width to length ratio of each rectangle. Then simplify the ratio.**



**Rewrite the fraction so that the numerator and denominator have the same units. Then simplify.**

13.  $\frac{2 \text{ yd}}{24 \text{ in.}}$

14.  $\frac{60 \text{ mm}}{1 \text{ cm}}$

15.  $\frac{40 \text{ g}}{1 \text{ kg}}$

16.  $\frac{20 \text{ ft}}{3 \text{ yd}}$

17.  $\frac{3 \text{ lb}}{12 \text{ oz}}$

18.  $\frac{5 \text{ weeks}}{30 \text{ days}}$

19.  $\frac{85 \text{ cm}}{0.5 \text{ m}}$

20.  $\frac{2 \text{ mi}}{60 \text{ ft}}$

**Solve the proportion.**

21.  $\frac{x}{3} = \frac{10}{15}$

22.  $\frac{y}{10} = \frac{2}{5}$

23.  $\frac{20}{30} = \frac{m}{120}$

24.  $\frac{4}{x+2} = \frac{16}{x+5}$

25.  $\frac{3}{y-2} = \frac{15}{y}$

26.  $\frac{2}{y-3} = \frac{3}{y}$

27. On an N-gauge model train set, a tank car is 3.75 inches long. An actual tank car is 50 feet long. What is the ratio of the length of the actual car to the length of the model tank car?



**Practice B**

For use with pages 457–464

**Simplify the ratio.**

1.  $\frac{8 \text{ books}}{24 \text{ books}}$

2.  $\frac{24 \text{ trees}}{14 \text{ trees}}$

3.  $\frac{18 \text{ balls}}{36 \text{ balls}}$

4.  $\frac{48 \text{ feet}}{36 \text{ feet}}$

**Rewrite the fraction so that the numerator and denominator have the same units. Then simplify.**

5.  $\frac{2 \text{ qt}}{4 \text{ gal}}$

6.  $\frac{250 \text{ mg}}{10 \text{ g}}$

7.  $\frac{24 \text{ oz}}{2 \text{ lb}}$

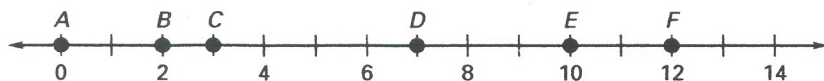
8.  $\frac{14 \text{ ft}}{6 \text{ yd}}$

9.  $\frac{4 \text{ ft}}{8 \text{ in.}}$

10.  $\frac{4 \text{ days}}{36 \text{ hours}}$

11.  $\frac{1.5 \text{ m}}{80 \text{ cm}}$

12.  $\frac{440 \text{ yd}}{2 \text{ mi}}$

**Use the number line to find the ratio of the distances.**

13.  $\frac{AB}{CD} = ?$

14.  $\frac{BC}{DE} = ?$

15.  $\frac{AC}{BD} = ?$

16.  $\frac{CF}{AB} = ?$

**Solve the proportion.**

17.  $\frac{x}{6} = \frac{9}{24}$

18.  $\frac{y}{9} = \frac{4}{6}$

19.  $\frac{17}{24} = \frac{m}{120}$

20.  $\frac{6}{x} = \frac{8}{x+3}$

21.  $\frac{4}{y+3} = \frac{3}{y-4}$

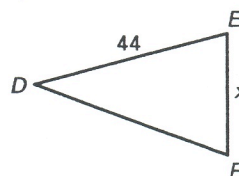
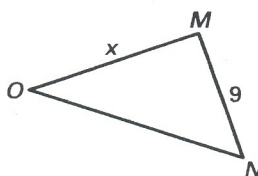
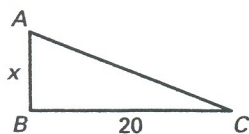
22.  $\frac{5}{2y-7} = \frac{3}{y}$

**The ratio of two side lengths of the triangle is given. Solve for the variable.**

23.  $AB : BC$  is 2:5

24.  $MN : MO$  is 3:4

25.  $DE : EF$  is 8:5

**In Exercises 26 and 27, use the following information.**

The largest submarines in the United States Navy are of the Ohio class. Each submarine is 560 feet long.

26. You purchase a scale model of one of the submarines. The package states the scale of 1 inch : 16 feet. What is the length of the completed model?

27. If the model is approximately 5 inches tall, what is the height of the actual submarine?

**Skills Practice**

8.1 Blue

**Using Ratios and Proportions***Write each ratio in simplest form.*

1.  $\frac{15}{20}$

2.  $\frac{7}{49}$

3.  $\frac{10}{15}$

4.  $\frac{28}{35}$

5.  $\frac{11}{22}$

6.  $\frac{20}{25}$

7.  $\frac{3}{15}$

8.  $\frac{18}{81}$

9.  $\frac{36}{27}$

10.  $\frac{55}{33}$

11.  $\frac{12}{2}$

12.  $\frac{40}{25}$

13. 15 feet to 25 feet

14. 48 centimeters to 15 centimeters

15. 45 meters to 60 meters

16. 14 inches to 24 inches

17. 12 inches to 3 feet

18. 80 centimeters to 2 meters

19. 4 feet to 10 yards

20. 8 quarts to 5 gallons

*Solve each proportion.*

21.  $\frac{3}{8} = \frac{6}{x}$

22.  $\frac{24}{18} = \frac{x}{3}$

23.  $\frac{7}{12} = \frac{14}{x}$

24.  $\frac{8}{28} = \frac{x}{21}$

25.  $\frac{4}{8} = \frac{x}{12}$

26.  $\frac{32}{6} = \frac{16}{x}$

27.  $\frac{9}{5} = \frac{x}{20}$

28.  $\frac{5}{x} = \frac{35}{70}$

29.  $\frac{22}{18} = \frac{x}{27}$

30.  $\frac{2}{x} = \frac{14}{21}$

31.  $\frac{8}{x} = \frac{56}{7}$

32.  $\frac{x}{5} = \frac{6}{30}$